**Number System**

**Divisibility**

**1) Which one of the following numbers is divisible by 99?**

**a) 3572404 b) 135792 c) 913464 d) 114345**

**Solution: c) & d)**

To determine whether a number is divisible by 99, we need to check if it is divisible by both 9 and 11 since 99 = 9 \* 11.

To check for divisibility by 9, we sum up the digits of the number and see if the result is divisible by 9.

To check for divisibility by 11, we alternate adding and subtracting the digits from left to right. If the result is divisible by 11, then the original number is also divisible by 11.

Let's apply this to the given numbers:

a) 3572404 Sum of digits: 3 + 5 + 7 + 2 + 4 + 0 + 4 = 25 The sum (25) is not divisible by 9, so the number is not divisible by 99.

b) 135792 Sum of digits: 1 + 3 + 5 + 7 + 9 + 2 = 27 The sum (27) is divisible by 9, but let's check for divisibility by 11: 1 - 3 + 5 - 7 + 9 - 2 = 3 (which is not divisible by 11) So, the number is not divisible by 99.

c) 913464 Sum of digits: 9 + 1 + 3 + 4 + 6 + 4 = 27 The sum (27) is divisible by 9, and let's check for divisibility by 11: 9 - 1 + 3 - 4 + 6 - 4 = 9 (which is divisible by 11) Thus, the number is divisible by 99.

d) 114345 Sum of digits: 1 + 1 + 4 + 3 + 4 + 5 = 18 The sum (18) is divisible by 9, but let's check for divisibility by 11: 1 - 1 + 4 - 3 + 4 - 5 = 0 (which is divisible by 11) So, the number is divisible by 99.

Therefore, the numbers that are divisible by 99 are: c) 913464 d) 114345

**2) If n is an integer, what is the remainder when (2n + 2)2 is divided by 4?**

**Solution:**

Let's simplify the expression (2n + 2) \* (2n + 2) and find the remainder when it's divided by 4.

(2n + 2) \* (2n + 2) = 4n^2 + 4n + 4

To find the remainder when this expression is divided by 4, we need to look at the last two terms (4n + 4), as the term 4n^2 is already divisible by 4 (since it has a factor of 4).

Now, let's consider the possibilities for n:

1. If n is even (n = 2k, where k is an integer), then 4n is also even (divisible by 4), and the remainder when divided by 4 will be 0.
2. If n is odd (n = 2k + 1, where k is an integer), then 4n is also odd, and when an odd number is divided by 4, the remainder is 1.

So, the remainder when (2n + 2) \* (2n + 2) is divided by 4 depends on whether n is even or odd:

* If n is even, the remainder is 0.
* If n is odd, the remainder is 1.

**3) Find two nearest numbers to 19506 which are divisible by 9?**

Solution:

To find the nearest numbers to 19506 that are divisible by 9, we can start by finding the largest multiple of 9 that is less than or equal to 19506 and then find the smallest multiple of 9 that is greater than 19506.

Step 1: Largest multiple of 9 less than or equal to 19506. Divide 19506 by 9 to find the quotient: 19506 ÷ 9 = 2167 with a remainder of 3.

The largest multiple of 9 less than or equal to 19506 is 9 \* 2167 = 19503.

Step 2: Smallest multiple of 9 greater than 19506. Add 1 to the quotient from Step 1 to find the next multiple of 9: 2167 + 1 = 2168.

The smallest multiple of 9 greater than 19506 is 9 \* 2168 = 19512.

So, the two nearest numbers to 19506 that are divisible by 9 are 19503 and 19512.

**4) What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?**

**Solution:**

For a number to be divisible by 8, the last three digits of the number must be divisible by 8.

For a number to be divisible by 11, the difference between the sum of the digits in odd positions (1st, 3rd, 5th, etc.) and the sum of the digits in even positions (2nd, 4th, 6th, etc.) must be divisible by 11.

Let's apply these rules to the number M39048458N:

1. Divisibility by 8: The last three digits are "458." To check if it's divisible by 8, we need to see if 458 is divisible by 8. Since 458 is not divisible by 8, the entire number M39048458N is not divisible by 8. Therefore, we can stop here as there are no valid values for M and N.

The number M39048458N is not divisible by 8 and 11, so there are no single-digit integer values for M and N that satisfy the divisibility conditions.

**5) How many pairs of X and Y are possible in the number 763X4Y2, if the number is divisible by 9?**

**Solution:**

To determine the possible pairs of X and Y for the number 763X4Y2 to be divisible by 9, we need to apply the divisibility rule for 9.

A number is divisible by 9 if the sum of its digits is divisible by 9. Let's find the sum of the digits in the number 763X4Y2:

Sum = 7 + 6 + 3 + X + 4 + Y + 2

Since we are looking for single-digit integers for X and Y, the possible values for X and Y are 0 to 9.

To make the entire number divisible by 9, we need to find pairs of X and Y that make the sum of the digits divisible by 9. The possible values for X and Y are:

1. X = 0, Y = 9: Sum = 7 + 6 + 3 + 0 + 4 + 9 + 2 = 31 (Not divisible by 9)
2. X = 1, Y = 8: Sum = 7 + 6 + 3 + 1 + 4 + 8 + 2 = 31 (Not divisible by 9)
3. X = 2, Y = 7: Sum = 7 + 6 + 3 + 2 + 4 + 7 + 2 = 31 (Not divisible by 9)
4. X = 3, Y = 6: Sum = 7 + 6 + 3 + 3 + 4 + 6 + 2 = 31 (Not divisible by 9)
5. X = 4, Y = 5: Sum = 7 + 6 + 3 + 4 + 4 + 5 + 2 = 31 (Not divisible by 9)
6. X = 5, Y = 4: Sum = 7 + 6 + 3 + 5 + 4 + 4 + 2 = 31 (Not divisible by 9)
7. X = 6, Y = 3: Sum = 7 + 6 + 3 + 6 + 4 + 3 + 2 = 31 (Not divisible by 9)
8. X = 7, Y = 2: Sum = 7 + 6 + 3 + 7 + 4 + 2 + 2 = 31 (Not divisible by 9)
9. X = 8, Y = 1: Sum = 7 + 6 + 3 + 8 + 4 + 1 + 2 = 31 (Not divisible by 9)
10. X = 9, Y = 0: Sum = 7 + 6 + 3 + 9 + 4 + 0 + 2 = 31 (Not divisible by 9)

After checking all possible pairs, we find that none of the pairs of X and Y make the number 763X4Y2 divisible by 9. Therefore, there are no pairs of X and Y that satisfy the divisibility condition.

**6) When the integer n is divided by 8, the remainder is 3. What is the remainder if 6n is divided by 8?**

**Solution:**

Let's use the given information to find the remainder when 6n is divided by 8.

We know that when the integer n is divided by 8, the remainder is 3. Mathematically, we can represent this as:

n ≡ 3 (mod 8)

This means n is of the form 8k + 3, where k is an integer.

Now, we want to find the remainder when 6n is divided by 8. Mathematically, we can represent this as:

6n ≡ ? (mod 8)

Substitute n = 8k + 3 into the expression for 6n:

6n = 6(8k + 3) = 48k + 18

Now, we need to find the remainder when 48k + 18 is divided by 8. To do this, we can first simplify the expression:

48k + 18 = 8(6k + 2) + 2

Now, we can see that 48k + 18 can be written as 8 times some integer plus 2. Therefore, the remainder when 48k + 18 is divided by 8 is 2.

So, the remainder when 6n is divided by 8 is 2.

**7) If the product 4864 x 9P2 is divisible by 12, then what is the value of P?**

Solution:

To check if the product 4864 x 9P2 is divisible by 12, we need to check if it's divisible by both 3 and 4 since 12 = 3 x 4.

1. Divisibility by 3: A number is divisible by 3 if the sum of its digits is divisible by 3. Let's find the sum of the digits of 9P2: 9 + P + 2 = 11 + P

For the entire product to be divisible by 3, the sum 11 + P must be divisible by 3. The possible values for P that make the sum divisible by 3 are P = 1 and P = 4.

1. Divisibility by 4: A number is divisible by 4 if its last two digits form a number divisible by 4. For the number 9P2, we need to check if 92, 12, 42, and 72 are divisible by 4.

Checking the divisibility by 4 for each case:

* 92 is not divisible by 4.
* 12 is divisible by 4.
* 42 is not divisible by 4.
* 72 is divisible by 4.

Only two possibilities satisfy both conditions: P = 1 (92 is not divisible by 4) and P = 4 (72 is divisible by 4).

Therefore, the possible values for P are P = 1 and P = 4.

**8) If the number 7X86038 is exactly divisible by 11, then the smallest whole number in place of X?**

Solution:

To check if the number 7X86038 is divisible by 11, we can use the divisibility rule for 11:

A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is either 0 or divisible by 11.

The number 7X86038 can be split into odd and even position digits as follows: Odd positions: 7, 8, 0, 8 Even positions: X, 6, 3

Now, let's find the sum of the digits in odd positions and the sum of the digits in even positions:

Sum of digits in odd positions = 7 + 8 + 0 + 8 = 23 Sum of digits in even positions = X + 6 + 3 = X + 9

For the number to be divisible by 11, the difference between these two sums must be either 0 or divisible by 11:

23 - (X + 9) = 23 - X - 9 = 14 - X

Now, to find the smallest whole number for X that satisfies this condition, we need to find the factors of 14 that are either 0 or divisible by 11.

The only possible value for X that makes 14 - X divisible by 11 is X = 3.

Let's verify it:

23 - (3 + 9) = 23 - 12 = 11

Since 11 is divisible by 11, the number 7X86038 is divisible by 11 when X = 3.

Therefore, the smallest whole number in place of X is 3.

**9) If an integer n is divisible by 3, 5 and 12, what is the next larger integer divisible by all these numbers?**

**a) n2 b) n + 180 c) 2n d) n + 60**

Solution:

To find the next larger integer divisible by 3, 5, and 12, we need to find the least common multiple (LCM) of these three numbers.

The LCM of three numbers can be found by finding the LCM of the two of them first, and then finding the LCM of that result with the third number.

1. LCM(3, 5) = 15
2. LCM(15, 12) = 60

So, the LCM of 3, 5, and 12 is 60.

Now, we want to find the next larger integer divisible by all these numbers (n, in this case). To do that, we need to find the smallest multiple of 60 that is greater than n.

The next larger integer divisible by all three numbers is given by (n + 60).

Therefore, the correct answer is (d) n + 60.

**10) What is the product of the largest and the smallest possible values of M for which a number 5M83M4M1 is divisible by 9?**

Solution:

For a number to be divisible by 9, the sum of its digits must be divisible by 9.

Let's find the sum of the digits in the number 5M83M4M1:

Sum = 5 + M + 8 + 3 + M + 4 + M + 1 Sum = 13 + 3M

For the entire number to be divisible by 9, the sum 13 + 3M must be divisible by 9.

To make the sum divisible by 9, the value of M must be such that 3M is a multiple of 9. The possible values for M are 0, 3, 6, or 9.

1. If M = 0: Sum = 13 + 3(0) = 13 (Not divisible by 9)
2. If M = 3: Sum = 13 + 3(3) = 13 + 9 = 22 (Not divisible by 9)
3. If M = 6: Sum = 13 + 3(6) = 13 + 18 = 31 (Not divisible by 9)
4. If M = 9: Sum = 13 + 3(9) = 13 + 27 = 40 (Not divisible by 9)

None of the values of M make the sum divisible by 9.

Therefore, the number 5M83M4M1 is not divisible by 9 for any possible value of M.

As a result, there are no largest or smallest values of M, and the product of the largest and smallest possible values of M is undefined in this case.

**Unit digits (Cyclicity)**

**1) What is the unit digit in the product (365 x 659 x 771)?**

Solution:

To find the unit digit in the product of (3 multiplied 65 times) x (6 multiplied 59 times) x (7 multiplied 71 times), we only need to focus on the unit digits of each number and their pattern when multiplied.

1. The unit digit of 3^65: The unit digit of 3 when raised to any power follows the pattern {3, 9, 7, 1, 3, 9, 7, 1, ...}. Since 65 is one less than a multiple of 4 (65 = 4 \* 16 + 1), the unit digit of 3^65 is the fifth element in the pattern, which is 3.
2. The unit digit of 6^59: The unit digit of 6 when raised to any power follows the pattern {6, 6, 6, 6, ...}. Since 59 is not a multiple of 4, the unit digit of 6^59 remains 6.
3. The unit digit of 7^71: The unit digit of 7 when raised to any power follows the pattern {7, 9, 3, 1, 7, 9, 3, 1, ...}. Since 71 is three more than a multiple of 4 (71 = 4 \* 17 + 3), the unit digit of 7^71 is the fourth element in the pattern, which is 1.

Now, let's multiply the unit digits together:

Unit digit = 3 (from 3^65) x 6 (from 6^59) x 1 (from 7^71) = 3 x 6 x 1 = 18

The unit digit of the product is 8 (the unit digit of 18).

Therefore, the unit digit in the product of (3 multiplied 65 times) x (6 multiplied 59 times) x (7 multiplied 71 times) is 8.

**2) Find unit digit of product (173)^45\*(152)^77\*(777)^999?**

Solution:

To find the unit digit of the product (173)^45 \* (152)^77 \* (777)^999, we need to find the unit digit of each individual number and then multiply them together.

1. Unit digit of (173)^45: The unit digit of 173 is 3. When any number ending in 3 is raised to an odd power, the unit digit of the result will be 3 as well.
2. Unit digit of (152)^77: The unit digit of 152 is 2. When any number ending in 2 is raised to any positive power, the unit digit of the result will remain 2.
3. Unit digit of (777)^999: The unit digit of 777 is 7. When any number ending in 7 is raised to any power, the unit digit of the result will remain 7.

Now, let's multiply the unit digits together:

Unit digit = 3 (from (173)^45) \* 2 (from (152)^77) \* 7 (from (777)^999) = 3 \* 2 \* 7 = 42

The unit digit of the product (173)^45 \* (152)^77 \* (777)^999 is 2.

Therefore, the unit digit of the given product is 2.

**3) What is the unit's digit of the number 6^256–4^256?**

Solution:

To find the unit's digit of the number 6^256 - 4^256, we need to compute the individual unit's digits of 6^256 and 4^256, and then find the difference between them.

Let's start with 6^256: 6^1 = 6, 6^2 = 36, 6^3 = 216, 6^4 = 1296, ... The pattern for the unit's digit of 6 raised to any power is {6, 6, 6, 6, ...}. Since 256 is a multiple of 4 (256 = 4 \* 64), the unit's digit of 6^256 is 6.

Next, let's find the unit's digit of 4^256: 4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256, ... The pattern for the unit's digit of 4 raised to any even power is {6, 6, 6, 6, ...}, and for any odd power, it is {4, 4, 4, 4, ...}. Since 256 is a multiple of 4 (256 = 4 \* 64), the unit's digit of 4^256 is 6.

Now, we can find the difference between 6^256 and 4^256: 6^256 - 4^256 = 6 - 6 = 0

The unit's digit of the number 6^256 - 4^256 is 0.

**4) Find the unit's digit in 264^102+264^103?**

**Solution:**

To find the unit's digit in the sum 264^102 + 264^103, we need to consider the unit's digit of each term and then add them together.

Let's start with 264^102:

The unit's digit of 264^1 is 4. The unit's digit of 264^2 is 6. The unit's digit of 264^3 is 4. The unit's digit of 264^4 is 6. ... and so on.

We can see that the pattern for the unit's digit of 264 raised to any power is {4, 6, 4, 6, ...}.

Since 102 is even, the unit's digit of 264^102 is 6.

Now, let's find 264^103:

Since the unit's digit of 264^1 is 4 and the pattern for the unit's digit of 264 raised to any power is {4, 6, 4, 6, ...}, we can determine that the unit's digit of 264^103 is 4.

Finally, let's add the unit's digits together:

Unit digit = 6 (from 264^102) + 4 (from 264^103) = 10

The unit's digit of the sum 264^102 + 264^103 is 0.

Therefore, the unit's digit in the given sum is 0.

**5) What is the unit digit of (316)^3n + 1?**

Solution:

The unit digit of (316)^3: The unit digit of 316 is 6. When any number ending in 6 is raised to any positive power, the unit digit of the result follows the pattern {6, 6, 6, 6, ...}.

Now, let's consider (316)^3n. Since (316)^3 follows the pattern of unit digit 6, when we raise it to any positive power n, the unit digit of the result will still be 6.

Next, we add 1 to the unit digit of (316)^3n, which is 6:

6 + 1 = 7

Therefore, the unit digit of the expression (316)^3n + 1 is 7.

**6) What is the unit digit in (7^95- 3^58)?**

Solution:

To find the unit digit in the expression (7^95 - 3^58), we need to compute the individual unit digits of 7^95 and 3^58, and then find their difference.

Let's start with 7^95:

The unit digit of 7^1 is 7. The unit digit of 7^2 is 9. The unit digit of 7^3 is 3. The unit digit of 7^4 is 1. ... and so on.

We can see that the pattern for the unit's digit of 7 raised to any power is {7, 9, 3, 1, 7, 9, 3, 1, ...}.

Since 95 is one less than a multiple of 4 (95 = 4 \* 23 + 3), the unit digit of 7^95 is the third element in the pattern, which is 3.

Next, let's find the unit digit of 3^58:

The unit digit of 3^1 is 3. The unit digit of 3^2 is 9. The unit digit of 3^3 is 7. The unit digit of 3^4 is 1. ... and so on.

We can see that the pattern for the unit's digit of 3 raised to any power is {3, 9, 7, 1, 3, 9, 7, 1, ...}.

Since 58 is even, the unit digit of 3^58 is the second element in the pattern, which is 9.

Now, let's find the difference between 7^95 and 3^58:

7^95 - 3^58 = 3 - 9 = -6

Since we are looking for the unit digit, we take the positive value of -6, which is 6.

Therefore, the unit digit in the expression (7^95 - 3^58) is 6.

Top of Form

**7) What is the rightmost non-zero digit of the number 30^2720**

**Solution:**

To find the rightmost non-zero digit of the number 30^2720, we need to focus on the prime factors of the base number 30 (2 and 3) and the power 2720.

Since the rightmost non-zero digit only depends on the last digits of the number (the units digit), we can ignore the numbers raised to any power that don't affect the units digit.

Let's analyze the units digit of the powers of 3:

3^1 = 3 3^2 = 9 3^3 = 27 3^4 = 81 3^5 = 243 3^6 = 729 3^7 = 2187 3^8 = 6561

The units digit of powers of 3 follows a repeating pattern: {3, 9, 7, 1}.

Now, let's analyze the units digit of the powers of 2:

2^1 = 2 2^2 = 4 2^3 = 8 2^4 = 16 2^5 = 32 2^6 = 64 2^7 = 128 2^8 = 256

The units digit of powers of 2 follows a repeating pattern: {2, 4, 8, 6}.

Since we are interested in the rightmost non-zero digit of 30^2720, we only need to consider the units digits of 3^2720 and 2^2720.

The units digit of 3^2720 is the same as the units digit of 3^4 (since 2720 is a multiple of 4): 3^4 = 81, so the units digit is 1.

The units digit of 2^2720 is the same as the units digit of 2^4 (since 2720 is a multiple of 4): 2^4 = 16, so the units digit is 6.

Now, to find the rightmost non-zero digit of 30^2720, we multiply the rightmost non-zero digits of 3^2720 and 2^2720:

Rightmost non-zero digit = 1 \* 6 = 6

The rightmost non-zero digit of the number 30^2720 is 6.

**8) What will be the last digit of the number obtained by multiplying the numbers 81\*82\*83\*84\*86\*87\*88\*89?**

**Ans:**

To find the last digit of the number obtained by multiplying the numbers 81 \* 82 \* 83 \* 84 \* 86 \* 87 \* 88 \* 89, we can calculate the product and then look at the last digit of the result.

81 \* 82 \* 83 \* 84 \* 86 \* 87 \* 88 \* 89 = 3397327171264

The last digit of the product is 4.

Therefore, the last digit of the number obtained by multiplying these numbers is 4.

**9) Find the last three-digits of the product: 12345x54321**

Ans:

To find the last three digits of the product 12345 x 54321, we need to calculate the product and then extract the last three digits from the result.

12345 x 54321 = 670592745

The last three digits of the product are 745.

Therefore, the last three digits of the product 12345 x 54321 are 745.

**10) Find the last digit of 15+25+35……95**

Solution:

To find the last digit of the sum 15 + 25 + 35 + ... + 95, we can first identify the pattern and then add up the numbers.

The given series is an arithmetic sequence with a common difference of 10. We can express it as:

15, 25, 35, ..., 95

The last digit of each term in the sequence is:

5, 5, 5, ..., 5

Since the last digit is the same for every term, we can determine the last digit of the sum by considering the number of terms in the sequence.

There are a total of 9 terms in the sequence (from 15 to 95 with a common difference of 10). When we add up numbers with the same last digit (5) for any number of times, the last digit of the sum remains the same.

Thus, the last digit of the sum 15 + 25 + 35 + ... + 95 is 5.

**Factorials**

1) What is the highest power of 21 that divides 20!?

Ans:

To find the highest power of 21 that divides 20!, we need to count the number of factors of 21 in the prime factorization of 20!.

First, let's find the prime factorization of 21:

21 = 3 \* 7

Now, let's count the number of factors of 21 in the prime factorization of 20!:

20! = 20 × 19 × 18 × 17 × 16 × 15 × 14 × 13 × 12 × 11 × 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1

To find the number of factors of 21, we need to count the number of multiples of 21, which are present in the list above.

Factors of 21 in 20! are: 21, 2 \* 21, 3 \* 21, 4 \* 21, 5 \* 21, 6 \* 21, 7 \* 21, 8 \* 21, 9 \* 21, 10 \* 21, 11 \* 21, 12 \* 21, 13 \* 21, 14 \* 21, 15 \* 21, 16 \* 21, 17 \* 21, 18 \* 21, 19 \* 21, 20 \* 21

There are 20 multiples of 21 in the list above.

Therefore, the highest power of 21 that divides 20! is 21^20.

Note: We don't need to consider the powers of 21 in the prime factorization of 20! since there are only 20 multiples of 21 in the list, and each multiple contributes one power of 21.

2) What is the highest power of 32 that divides 31!?

Ans:

To find the highest power of 32 that divides 31!, we need to count the number of factors of 32 in the prime factorization of 31!.

First, let's find the prime factorization of 32:

32 = 2^5

Now, let's find the prime factorization of 31!:

31! = 31 × 30 × 29 × ... × 3 × 2 × 1

To find the number of factors of 32 in 31!, we need to count the number of times 32 can be formed as the product of 2 and another factor present in 31!. Since 32 = 2^5, we need to count the number of factors of 2^5 in 31!.

To do this, we need to count the multiples of 2 in the numbers 1 to 31. However, we should also consider that some of the numbers in the factorization of 31! contain more than one factor of 2.

Count of factors of 2 in 1 to 31:

1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14 × 15 × 16 × 17 × 18 × 19 × 20 × 21 × 22 × 23 × 24 × 25 × 26 × 27 × 28 × 29 × 30 × 31

In this list, the numbers 4, 8, 12, 16, 20, 24, 28, and 32 (which is not in the product but it's a power of 2) contribute two factors of 2 each. Additionally, 16, 24, and 32 contribute one extra factor of 2 since they have a power of 2 as one of their factors.

Therefore, the total count of factors of 2 in 31! is:

1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 + 1 = 15

Since 32 = 2^5, we can form 32 with five factors of 2.

Therefore, the highest power of 32 that divides 31! is 32^3 = 2^(5\*3) = 2^15.

3) Find the largest number less than 28 which divides 28!?

Ans:

To find the largest number less than 28 that divides 28!, we need to determine the highest power of 28 that appears in the prime factorization of 28!.

First, let's find the prime factorization of 28:

28 = 2^2 \* 7^1

Now, let's find the prime factorization of 28!:

28! = 28 × 27 × 26 × ... × 2 × 1

To determine the highest power of 28 that divides 28!, we need to count the number of factors of 28 (2^2 \* 7^1) in the prime factorization of 28!.

The factors of 28 in 28! are:

28, 2 \* 14, 4 \* 7

There are three multiples of 28 (2^2 \* 7^1) in 28!.

Therefore, the largest number less than 28 that divides 28! is 28.

4) Find the number of zeroes at the end of 97!

Ans:

To find the number of zeros at the end of 97!, we need to count the factors of 10 in the prime factorization of 97!.

A zero at the end of a number is obtained by the multiplication of 10, which can be expressed as 2 \* 5. So, we need to count the number of pairs of 2 and 5 in the prime factorization of 97!.

First, let's find the prime factorization of 97!:

97! = 97 × 96 × 95 × ... × 2 × 1

Now, let's determine the number of factors of 2 and 5 in the prime factorization:

Factors of 2: We have to count the multiples of 2 in the list above, which are 96, 94, 92, ..., 4, 2. We can observe that every second number in the list is a multiple of 2.

Number of factors of 2 in 97!: 97/2 = 48

Factors of 5: We have to count the multiples of 5 in the list above, which are 95, 90, 85, ..., 5. Every fifth number in the list is a multiple of 5.

Number of factors of 5 in 97!: 97/5 = 19

Now, we have to find the number of pairs of 2 and 5. Since we have 19 factors of 5, but 48 factors of 2, we can form 19 pairs of 2 and 5.

Therefore, the number of zeros at the end of 97! is 19.

5) What is the highest power of 12 that divides 54!?

Ans:

To find the highest power of 12 that divides 54!, we need to determine the number of factors of 12 (2^2 \* 3^1) in the prime factorization of 54!.

First, let's find the prime factorization of 54:

54 = 2 \* 3^3

Now, let's find the prime factorization of 54!:

54! = 54 × 53 × 52 × ... × 2 × 1

To determine the number of factors of 12 in 54!, we need to count the multiples of 12 (2^2 \* 3^1) in the list.

Factors of 12 in 54!: 12, 24, 36, 48

Now, let's count the number of times each of these multiples appears in the list:

* 12 appears once.
* 24 appears twice.
* 36 appears once.
* 48 appears once.

Now, we need to count the total number of factors of 12 (2^2 \* 3^1) in 54!. We have one factor of 12 for each of the multiples listed above:

Number of factors of 12 in 54! = 1 + 2 + 1 + 1 = 5

Therefore, the highest power of 12 (2^2 \* 3^1) that divides 54! is 12^5 = (2^2 \* 3^1)^5 = 2^(2*5) \* 3^(1*5) = 2^10 \* 3^5.

6) Find the least value of x such that 60!/2^x is an odd number.

Ans:

To find the least value of x such that 60!/2^x is an odd number, we need to determine the highest power of 2 in the prime factorization of 60!.

First, let's find the prime factorization of 60:

60 = 2^2 \* 3^1 \* 5^1

Now, let's find the prime factorization of 60!:

60! = 60 × 59 × 58 × ... × 2 × 1

To determine the highest power of 2 in 60!, we need to count the number of multiples of 2 in the list.

Number of multiples of 2 in 60! = 30 + 15 + 7 + 3 + 1 = 56

Since 60! contains 56 factors of 2, the value of x should be at least 56.

Therefore, the least value of x such that 60!/2^x is an odd number is 56.

7) Find the least value of ‘n’ if no factorial can have ‘n’ zeroes?

Ans:

To find the least value of 'n' such that no factorial can have 'n' zeroes, we need to understand how the number of trailing zeroes in a factorial is related to its prime factorization.

A trailing zero in a factorial is formed by the multiplication of 10, which is the product of 2 and 5. To count the number of trailing zeroes in the factorial of a number, we need to count the number of pairs of 2 and 5 in its prime factorization.

The number of factors of 2 is usually more than the number of factors of 5, so to count the number of trailing zeroes, we only need to count the number of factors of 5 in the prime factorization of the factorial.

For example, in 5! = 5 × 4 × 3 × 2 × 1, there is one factor of 5, which means one trailing zero. In 10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1, there are two factors of 5, so there are two trailing zeroes.

To find the least value of 'n' such that no factorial can have 'n' zeroes, we need to find the first factorial without any trailing zeroes. This happens when there are no factors of 5 in the prime factorization of the factorial.

The first factorial without any trailing zeroes is 4! = 4 × 3 × 2 × 1. In this case, there are no factors of 5, so there are no trailing zeroes.

Therefore, the least value of 'n' is 0.

8) What is the highest power of 7! dividing 50! completely.

Ans:

To find the highest power of 7! that divides 50! completely, we need to determine the number of factors of 7! in the prime factorization of 50!.

First, let's find the prime factorization of 7!:

7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 2^4 × 3^2 × 5 × 7

Now, let's find the prime factorization of 50!:

50! = 50 × 49 × 48 × ... × 2 × 1

To determine the number of factors of 7! in 50!, we need to count the multiples of 7! (2^4 × 3^2 × 5 × 7) in the list above.

Number of multiples of 7! in 50!: 50, 49, 48, ..., 7

We need to count the multiples of 7 (7 × 1), which are 49 and 42, and also the multiples of 7^2 (7 × 7), which is 49.

Thus, there are three multiples of 7! in 50!.

Therefore, the highest power of 7! that divides 50! completely is 7!^3 = (2^4 × 3^2 × 5 × 7)^3 = 2^(4*3) × 3^(2*3) × 5^3 × 7^3 = 2^12 × 3^6 × 5^3 × 7^3.

9) How many more trailing zeroes would 625! have than 624!?

Ans:

To find the number of more trailing zeroes that 625! has than 624!, we need to count the number of factors of 5 in the prime factorization of each factorial and then find the difference.

The number of trailing zeroes in a factorial is determined by the number of factors of 5 in its prime factorization. We can use the following formula to calculate the number of trailing zeroes in a factorial n!:

Number of trailing zeroes in n! = floor(n/5) + floor(n/25) + floor(n/125) + ...

Let's calculate the number of trailing zeroes in 625!:

Number of trailing zeroes in 625! = floor(625/5) + floor(625/25) + floor(625/125) + floor(625/625) = 125 + 25 + 5 + 1 = 156

Now, let's calculate the number of trailing zeroes in 624!:

Number of trailing zeroes in 624! = floor(624/5) + floor(624/25) + floor(624/125) + floor(624/625) = 124 + 24 + 4 + 0 = 152

To find the number of more trailing zeroes that 625! has than 624!, we subtract the number of trailing zeroes in 624! from the number of trailing zeroes in 625!:

156 - 152 = 4

Therefore, 625! has 4 more trailing zeroes than 624!.

10) Find the number of zeroes at the end of 1^1×2^2×3^3…..×100^100

Ans:

To find the number of zeroes at the end of the product 1^1 × 2^2 × 3^3 × ... × 100^100, we need to count the number of factors of 10 in the prime factorization of the product. A factor of 10 is formed by the multiplication of 2 and 5.

To count the number of factors of 10 in the product, we need to determine the number of pairs of 2 and 5 in the prime factorization of each term. Notice that there are more factors of 2 than factors of 5, so we only need to count the number of factors of 5 in each term.

1^1 has no factors of 5. 2^2 has no factors of 5. 3^3 has one factor of 5. ... 10^10 has one factor of 5.

Now, from 11^11 to 100^100, each term has one factor of 5.

So, the number of factors of 5 in the product is: 0 + 0 + 1 + 1 + 1 + ... + 1 (100 times)

The sum of 100 ones is 100.

Therefore, the number of zeroes at the end of the product 1^1 × 2^2 × 3^3 × ... × 100^100 is 100.

**Factors**

1) What is the number of prime factors in 6^4x8^6x10^8x14^10x22^12

Ans:

To find the number of prime factors in the expression 6^4 × 8^6 × 10^8 × 14^10 × 22^12, we first need to write each term in its prime factorization form.

1. Prime factorization of 6: 6 = 2 × 3
2. Prime factorization of 8: 8 = 2^3
3. Prime factorization of 10: 10 = 2 × 5
4. Prime factorization of 14: 14 = 2 × 7
5. Prime factorization of 22: 22 = 2 × 11

Now, let's write each term in its prime factorization form:

6^4 = (2 × 3)^4 = 2^4 × 3^4 8^6 = (2^3)^6 = 2^18 10^8 = (2 × 5)^8 = 2^8 × 5^8 14^10 = (2 × 7)^10 = 2^10 × 7^10 22^12 = (2 × 11)^12 = 2^12 × 11^12

Now, let's multiply all the prime factors together:

Prime factors in 6^4 × 8^6 × 10^8 × 14^10 × 22^12:

2^4 × 3^4 × 2^18 × 2^8 × 5^8 × 2^10 × 7^10 × 2^12 × 11^12

Now, we can combine the like terms:

2^(4 + 18 + 8 + 10 + 12) × 3^4 × 5^8 × 7^10 × 11^12

Simplifying the exponents:

2^52 × 3^4 × 5^8 × 7^10 × 11^12

The number of prime factors in the expression is 5, as there are five different prime numbers (2, 3, 5, 7, and 11) present in the expression.

2) N= a^4xb^3xc^7. Find the number of perfect square factors of N where a,b,c are three distinct prime numbers.

Ans:

To find the number of perfect square factors of N = a^4 × b^3 × c^7, we need to consider the powers of the primes a, b, and c.

A perfect square factor is a factor that can be represented as a perfect square. For example, if the power of a prime factor is even, then it can be represented as a perfect square.

The powers of a, b, and c are 4, 3, and 7, respectively. To find the number of perfect square factors, we need to count the possible combinations of even powers of these primes.

For a: There are three possible even powers: 0 (which represents a^0 = 1), 2 (which represents a^2), and 4 (which represents a^4). So, there are 3 perfect square factors of a.

For b: There are two possible even powers: 0 (which represents b^0 = 1) and 2 (which represents b^2). So, there are 2 perfect square factors of b.

For c: There are four possible even powers: 0 (which represents c^0 = 1), 2 (which represents c^2), 4 (which represents c^4), and 6 (which represents c^6). So, there are 4 perfect square factors of c.

Now, to find the total number of perfect square factors of N, we need to count all possible combinations of these even powers of a, b, and c:

Number of perfect square factors of N = 3 (perfect square factors of a) × 2 (perfect square factors of b) × 4 (perfect square factors of c) = 24.

Therefore, the number of perfect square factors of N = a^4 × b^3 × c^7, where a, b, and c are three distinct prime numbers, is 24.

3) How many factors of 12^3x30^4x35^2 are even numbers?

Ans:

To find the number of factors of the number 12^3 × 30^4 × 35^2 that are even, we need to determine the powers of 2 in the prime factorization of each term.

1. Prime factorization of 12: 12 = 2^2 × 3^1
2. Prime factorization of 30: 30 = 2^1 × 3^1 × 5^1
3. Prime factorization of 35: 35 = 5^1 × 7^1

Now, let's calculate the powers of 2 in each term:

Powers of 2 in 12^3: (2^2)^3 = 2^6

Powers of 2 in 30^4: (2^1)^4 = 2^4

Powers of 2 in 35^2: (5^1)^2 = 5^2 (There are no factors of 2 in this term)

Now, we need to find all possible combinations of these powers of 2. To get an even number as the factor, we need to include at least one power of 2 in the factorization.

Number of even factors = (1 + 6) × (1 + 4) × (1 + 0) = 7 × 5 × 1 = 35

Therefore, there are 35 factors of 12^3 × 30^4 × 35^2 that are even numbers.

4) If N=2^7×3^4, M= 2^4×3^2×5, then find the number of factors of N that are common with the factors of M.

Ans:

To find the number of factors that N = 2^7 × 3^4 and M = 2^4 × 3^2 × 5 have in common, we need to consider the common prime factors between N and M and then count the number of possible combinations.

The common prime factors between N and M are 2 and 3. To find the factors of N and M, we can consider the powers of 2 and 3 in their prime factorizations:

Prime factorization of N: N = 2^7 × 3^4 Prime factorization of M: M = 2^4 × 3^2 × 5

Now, let's consider the powers of 2 and 3 in N and M:

Powers of 2 in N: 7 Powers of 2 in M: 4

Powers of 3 in N: 4 Powers of 3 in M: 2

To find the number of factors that N and M have in common, we need to find the possible combinations of powers of 2 and 3 that both N and M share.

For the powers of 2, there are two possible combinations: 4 and 7.

For the powers of 3, there are three possible combinations: 2 and 4.

Now, to find the total number of factors that N and M have in common, we need to multiply the number of possible combinations of powers of 2 and powers of 3:

Number of factors in common = (number of combinations of powers of 2) × (number of combinations of powers of 3) = 2 × 3 = 6

Therefore, N = 2^7 × 3^4 and M = 2^4 × 3^2 × 5 have 6 factors in common.

5) N is the smallest number that has 5 factors. How many factors does (N - 1) have?

Ans:

6) If both 11^2 and 3^4 are factors of the number Ax4^3x6^2x13^11, then what is the smallest possible value of A?

Ans:

Let's find the smallest number N that has 5 factors.

A number with 5 factors can be written in the form p^4, where p is a prime number. This is because a number with 5 factors can only have two distinct prime factors (p^4 = p \* p \* p \* p, with the possibility of p^2 \* q^2, but that would be larger than p^4 and not the smallest number).

Now, let's find the smallest prime number (p) that results in a number with 5 factors:

p = 2

The smallest number N with 5 factors is 2^4 = 16.

Now, let's find (N - 1):

(N - 1) = 16 - 1 = 15

To find the number of factors of (N - 1), we need to consider its prime factorization.

Prime factorization of 15: 15 = 3 \* 5

The number of factors of 15 can be found by counting the possible combinations of its prime factors: (1, 3, 5, 15)

So, (N - 1) = 15 has 4 factors: 1, 3, 5, and 15.

Therefore, (N - 1) has 4 factors.

7) Find the total number of factors of 10!

Ans:

To find the total number of factors of 10!, we need to calculate the prime factorization of 10! and then count the number of factors.

First, let's find the prime factorization of 10:

10 = 2 × 5

Now, let's find the prime factorization of 10!:

10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1

To find the number of factors of 10!, we need to count all the possible combinations of the prime factors in its prime factorization.

The prime factorization of 10! is:

10! = 2^8 × 3^4 × 5^2

To find the number of factors, we consider all possible combinations of the powers of 2, 3, and 5:

Number of factors = (8 + 1) × (4 + 1) × (2 + 1) = 9 × 5 × 3 = 135

Therefore, 10! has a total of 135 factors.

8) How many factors of 2^7x3^6x5^4x7^3 are even perfect squares?

Ans:

To find the number of factors of 2^7 × 3^6 × 5^4 × 7^3 that are even perfect squares, we need to consider the powers of the prime factors 2, 3, 5, and 7.

The factors of a number are formed by taking different combinations of the prime factors raised to different powers. A factor is a perfect square if all the powers of its prime factors are even.

The given number is 2^7 × 3^6 × 5^4 × 7^3. To form an even perfect square factor, we can choose even powers for each prime factor.

For 2, the possible even powers are 0, 2, 4, 6, and 7 (we can't choose 8 since the exponent is 7).

For 3, the possible even powers are 0, 2, 4, and 6 (we can't choose 8 since the exponent is 6).

For 5, the possible even powers are 0, 2, and 4 (we can't choose 6 since the exponent is 4).

For 7, the possible even powers are 0, 2 (we can't choose 4 since the exponent is 3).

Now, let's find all possible combinations of these even powers for each prime factor:

For 2: 5 possibilities (0, 2, 4, 6, 7) For 3: 4 possibilities (0, 2, 4, 6) For 5: 3 possibilities (0, 2, 4) For 7: 2 possibilities (0, 2)

To find the total number of even perfect square factors, we multiply the number of possibilities for each prime factor:

Number of even perfect square factors = 5 × 4 × 3 × 2 = 120

Therefore, there are 120 factors of 2^7 × 3^6 × 5^4 × 7^3 that are even perfect squares.

9) In how many ways can 480 be written as a product of two natural numbers?

Ans:

To find the number of ways 480 can be written as a product of two natural numbers, we need to consider all the possible pairs of natural numbers whose product is equal to 480.

Let's find the prime factorization of 480:

480 = 2^5 × 3^1 × 5^1

Now, let's list all the possible pairs of natural numbers whose product is 480:

1. 1 × 480 = 480
2. 2 × 240 = 480
3. 3 × 160 = 480
4. 4 × 120 = 480
5. 5 × 96 = 480
6. 6 × 80 = 480
7. 8 × 60 = 480
8. 10 × 48 = 480
9. 12 × 40 = 480
10. 15 × 32 = 480
11. 16 × 30 = 480
12. 20 × 24 = 480
13. 24 × 20 = 480
14. 30 × 16 = 480
15. 32 × 15 = 480
16. 40 × 12 = 480
17. 48 × 10 = 480
18. 60 × 8 = 480
19. 80 × 6 = 480
20. 96 × 5 = 480
21. 120 × 4 = 480
22. 160 × 3 = 480
23. 240 × 2 = 480
24. 480 × 1 = 480

There are a total of 24 ways to write 480 as a product of two natural numbers.

10) How many factors of 2^5x3^4x5^3 are not the factors of 2^3x5^4x7^5

Ans:

To find the factors of 2^5 × 3^4 × 5^3 that are not factors of 2^3 × 5^4 × 7^5, we need to determine the common factors and then subtract them from the total number of factors of 2^5 × 3^4 × 5^3.

First, let's find the prime factorization of both numbers:

2^5 × 3^4 × 5^3 = 32 × 81 × 125 2^3 × 5^4 × 7^5 = 8 × 625 × 16,807

Now, let's list all the factors of 2^5 × 3^4 × 5^3:

Factors: 1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100, 125, 160, 200, 250, 320, 400, 500, 625, 800, 1,000, 1,250, 1,600, 2,000, 2,500, 3,125, 4,000, 5,000, 6,250, 8,000, 10,000, 12,500, 16,000, 20,000, 25,000, 31,250, 40,000, 50,000, 62,500, 80,000, 100,000, 125,000, 160,000, 200,000, 250,000, 312,500, 400,000, 500,000, 625,000, 1,000,000, 1,250,000, 2,500,000, 5,000,000, 10,000,000

Now, let's list all the factors of 2^3 × 5^4 × 7^5:

Factors: 1, 2, 4, 5, 7, 8, 10, 14, 16, 20, 25, 28, 35, 40, 49, 50, 56, 70, 80, 98, 100, 112, 140, 175, 196, 200, 245, 280, 350, 392, 400, 490, 560, 700, 784, 875, 980, 1,000, 1,225, 1,400, 1,750, 1,960, 2,000, 2,450, 2,800, 3,500, 3,920, 4,375, 4,900, 5,000, 6,125, 7,000, 7,875, 8,750, 9,800, 12,250, 14,000, 15,625, 17,500, 19,600, 24,500, 28,000, 31,250, 35,000, 39,200, 48,125, 49,000, 62,500, 70,000, 78,125, 97,000, 98,000, 122,500, 140,000, 156,250, 195,000, 196,000, 245,000, 280,000, 312,500, 390,000, 392,000, 490,000, 625,000, 780,000, 980,000, 1,225,000, 1,250,000, 1,560,000, 1,960,000, 2,450,000, 2,500,000, 3,125,000, 3,900,000, 4,900,000, 6,250,000, 7,800,000, 9,800,000, 12,500,000, 15,625,000, 19,600,000, 24,500,000, 31,250,000, 39,200,000, 49,000,000, 78,125,000, 98,000,000, 122,500,000, 196,000,000, 245,000,000, 490,000,000, 980,000,000, 1,225,000,000, 2,450,000,000, 4,900,000,000, 9,800,000,000, 19,600,000,000

Now, let's find the common factors in both lists:

Common factors: 1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100, 125, 160, 200, 250, 320, 400, 500, 625, 800, 1,000, 1,250, 2,000, 2,500, 4,000, 5,000, 8,000, 10,000, 20,000, 25,000, 40,000, 50,000, 100,000, 125,000, 200,000, 250,000, 500,000, 1,000,000, 2,500,000, 5,000,000, 10,000,000

To find the factors that are not common, we need to subtract the number of common factors from the total number of factors in the first list:

Total factors in 2^5 × 3^4 × 5^3 = 6 × 5 × 4 = 120

Number of factors in common = 60 (which are the common factors listed above)

Number of factors not in common = 120 - 60 = 60

Therefore, there are 60 factors of 2^5 × 3^4 × 5^3 that are not factors of 2^3 × 5^4 × 7^5.

**Remainders**

1) What is the remainder when 725 is divided by 6?

725 ÷ 6 = 120 with a remainder of 5.

Therefore, the remainder when 725 is divided by 6 is 5.

2) What is the remainder when 345 is divided by 8?

345 ÷ 8 = 43 with a remainder of 1.

Therefore, the remainder when 345 is divided by 8 is 1.

3) Find the remainder when 496 is divided by 6.

496 ÷ 6 = 82 with a remainder of 4.

Therefore, the remainder when 496 is divided by 6 is 4.

4) What is the remainder when 141516 is divided by 5?

To find the remainder when 141516 is divided by 5, we sum up the digits of the number and then find the remainder of the sum when divided by 5.

1 + 4 + 1 + 5 + 1 + 6 = 18

Now, we find the remainder of 18 when divided by 5:

18 ÷ 5 = 3 with a remainder of 3.

Therefore, the remainder when 141516 is divided by 5 is 3.

5) Find the remainder when 6799 is divided by 7.

To find the remainder when 6799 is divided by 7, we can use the concept of modular arithmetic.

The remainder when a number is divided by 7 is the same as the remainder when the sum of its digits is divided by 7.

So, let's sum up the digits of 6799:

6 + 7 + 9 + 9 = 31

Now, we find the remainder of 31 when divided by 7:

31 ÷ 7 = 4 with a remainder of 3.

Therefore, the remainder when 6799 is divided by 7 is 3.

6) What is the remainder when 73x75x78x57x197x37 is divided by 34.

To find the remainder when 73 × 75 × 78 × 57 × 197 × 37 is divided by 34, we can use the property of modular arithmetic that states the remainder of a product is the same as the product of the remainders of each factor.

First, find the remainder of each factor when divided by 34:

1. 73 ÷ 34 = 2 with a remainder of 5.
2. 75 ÷ 34 = 2 with a remainder of 7.
3. 78 ÷ 34 = 2 with a remainder of 10.
4. 57 ÷ 34 = 1 with a remainder of 23.
5. 197 ÷ 34 = 5 with a remainder of 27.
6. 37 ÷ 34 = 1 with a remainder of 3.

Now, calculate the product of the remainders:

5 × 7 × 10 × 23 × 27 × 3 = 567,630

Next, find the remainder of 567,630 when divided by 34:

567,630 ÷ 34 = 16,687 with a remainder of 18.

Therefore, the remainder when 73 × 75 × 78 × 57 × 197 × 37 is divided by 34 is 18.

7) Let N = 1421\*1423\*1425. What is the remainder when N is divided by 12?

To find the remainder when N = 1421 × 1423 × 1425 is divided by 12, we can find the remainder of each factor when divided by 12 and then find the remainder of the product.

Let's find the remainder of each factor when divided by 12:

1. 1421 ÷ 12 = 118 with a remainder of 5.
2. 1423 ÷ 12 = 118 with a remainder of 7.
3. 1425 ÷ 12 = 118 with a remainder of 9.

Now, calculate the product of the remainders:

5 × 7 × 9 = 315

Finally, find the remainder of 315 when divided by 12:

315 ÷ 12 = 26 with a remainder of 3.

Therefore, the remainder when N = 1421 × 1423 × 1425 is divided by 12 is 3.

8) Find the remainder when 2256 is divided by 17.

To find the remainder when 2256 is divided by 17, we can use the concept of modular arithmetic.

First, let's check if there's a pattern in the remainders when the powers of 2 are divided by 17:

2^1 ÷ 17 = 0 with a remainder of 2 2^2 ÷ 17 = 0 with a remainder of 4 2^3 ÷ 17 = 0 with a remainder of 8 2^4 ÷ 17 = 1 with a remainder of 16 2^5 ÷ 17 = 3 with a remainder of 2 2^6 ÷ 17 = 6 with a remainder of 4 2^7 ÷ 17 = 13 with a remainder of 8 2^8 ÷ 17 = 27 with a remainder of 16 2^9 ÷ 17 = 55 with a remainder of 2

As we can see, the remainders follow a pattern: 2, 4, 8, 16, 2, 4, 8, 16, ...

The pattern repeats every 4 powers of 2. Since 2256 is divisible by 4 (2256 = 4 × 564), the remainder when 2256 is divided by 17 is the same as the remainder when 2^4 is divided by 17.

2^4 ÷ 17 = 1 with a remainder of 16

Therefore, the remainder when 2256 is divided by 17 is 16.

9) The remainder of 3997! /40 is:

a) 39 b) 0 c) 1 d) None of these

Ans:

To find the remainder of 3997! divided by 40, we can first simplify the expression by factoring 40.

40 can be factored as 2^3 \* 5.

Now, let's consider the prime factorization of 3997!:

3997! = 2^p \* 5^q \* ...

where p and q are the powers of 2 and 5, respectively, in the prime factorization of 3997!.

To find p and q, we can use the formula for the highest power of a prime p that divides n!:

p^q = ⌊n/p⌋ + ⌊n/p^2⌋ + ⌊n/p^3⌋ + ...

where ⌊x⌋ denotes the greatest integer less than or equal to x.

For p = 2 and n = 3997, we have:

q = ⌊3997/2⌋ + ⌊3997/2^2⌋ + ⌊3997/2^3⌋ + ... = 1998 + 999 + 499 + ... = 3991

For p = 5 and n = 3997, we have:

q = ⌊3997/5⌋ + ⌊3997/5^2⌋ + ⌊3997/5^3⌋ + ... = 799 + 159 + 31 + ... = 994

Now, the maximum power of 2 that divides 3997! is 2^3991, and the maximum power of 5 that divides 3997! is 5^994.

Since 40 = 2^3 \* 5, the highest power of 2 and 5 that can be divided from 3997! is 2^3 \* 5 = 40.

So, the remainder of 3997! divided by 40 is:

3997! / 40 ≡ 1 (mod 40)

Therefore, the remainder is 1 (option c).

10) Find the remainder on dividing 1!+2!+3!.....+100! by 7?

To find the remainder when 1! + 2! + 3! + ... + 100! is divided by 7, we need to calculate the sum and then find the remainder.

First, let's find the factorials from 1! to 100!:

1! = 1 2! = 2 3! = 6 4! = 24 5! = 120 6! = 720 7! = 5,040 8! = 40,320 9! = 362,880 10! = 3,628,800 ... (and so on)

Since 7! and all subsequent factorials are multiples of 7, we can ignore them in the sum, as they won't contribute to the remainder when divided by 7.

Now, let's consider the terms that are not multiples of 7:

1! + 2! + 3! + 4! + 5! + 6!

1! = 1 2! = 2 3! = 6 4! = 24 5! = 120 6! = 720

Sum = 1 + 2 + 6 + 24 + 120 + 720 = 873

Now, let's find the remainder of 873 when divided by 7:

873 ÷ 7 = 124 with a remainder of 5.

Therefore, the remainder when 1! + 2! + 3! + ... + 100! is divided by 7 is 5.

**HCF/LCM**

1) The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

a) 9000 b) 9400 c) 9600 d) 9800

Ans:

To find the greatest number of four digits that is divisible by 15, 25, 40, and 75, we need to find the least common multiple (LCM) of these four numbers.

First, let's find the prime factorization of each number:

15 = 3 × 5 25 = 5^2 40 = 2^3 × 5 75 = 3 × 5^2

Now, let's find the LCM by taking the highest power of each prime factor that appears in any of the numbers:

LCM = 2^3 × 3 × 5^2 = 8 × 3 × 25 = 600

The greatest number of four digits that is divisible by 15, 25, 40, and 75 is the largest multiple of 600 that is less than 10000.

10000 ÷ 600 = 16 with a remainder of 400

So, the largest multiple of 600 less than 10000 is 16 × 600 = 9600.

Therefore, the correct answer is option c) 9600.

2) The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is:

a) 279 b) 283 c) 308 d) 318

Ans:

To find the other number when the H.C.F. (highest common factor) is 11 and the L.C.M. (least common multiple) is 7700, and one of the numbers is 275, we can use the formula:

H.C.F. × L.C.M. = Product of the two numbers

Let the other number be x.

Given: H.C.F. = 11 L.C.M. = 7700 One number = 275

Using the formula, we have:

11 × 7700 = 275 × x

Now, solve for x:

x = (11 × 7700) / 275

x = 308

Therefore, the other number is 308 (option c).

3) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together including the toll at start?

a) 4 b) 10 c) 15 d) 16

Ans:

To find the number of times the bells toll together in 30 minutes, we need to find the LCM (Least Common Multiple) of the time intervals at which each bell tolls.

The time intervals are: 2, 4, 6, 8, 10, and 12 seconds.

To calculate the LCM, we first find the prime factorization of each interval:

2 = 2 4 = 2^2 6 = 2 × 3 8 = 2^3 10 = 2 × 5 12 = 2^2 × 3

Now, we take the highest powers of each prime factor that appear in any of the intervals to calculate the LCM:

LCM = 2^3 × 3 × 5 = 120 seconds

Now, we need to convert 30 minutes to seconds:

30 minutes = 30 × 60 seconds = 1800 seconds

Next, we find how many times the bells toll together within 1800 seconds (30 minutes) with an interval of 120 seconds:

Number of times = 1800 seconds / 120 seconds = 15 times

However, we need to include the toll at the start, so we add 1:

Total number of times = 15 + 1 = 16 times

Therefore, the bells toll together 16 times in 30 minutes, including the toll at the start (option d).

4) Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is:

a) 4 b) 5 c) 6 d) 8

Ans:

To find the greatest number N that will divide 1305, 4665, and 6905, leaving the same remainder in each case, we need to find the H.C.F. (highest common factor) of these numbers.

Let's find the remainders when each of these numbers is divided by the H.C.F.:

1305 ÷ H.C.F. = r 4665 ÷ H.C.F. = r 6905 ÷ H.C.F. = r

Since the remainder is the same in each case, the H.C.F. will be a divisor of the differences between these numbers. So, we can find the H.C.F. by calculating:

H.C.F. = GCD (1305 - 4665, 4665 - 6905)

H.C.F. = GCD (−3360, −2240)

Now, taking the absolute values:

H.C.F. = GCD (3360, 2240)

To find the GCD (greatest common divisor) of 3360 and 2240, we can use the Euclidean algorithm or prime factorization method.

Prime factorization of 3360: 3360 = 2^5 × 3 × 5 × 7

Prime factorization of 2240: 2240 = 2^5 × 5 × 7

To find the GCD, we take the product of the common prime factors raised to their lowest power:

GCD (3360, 2240) = 2^5 × 5 × 7 = 2240

So, the greatest number N that will divide 1305, 4665, and 6905, leaving the same remainder in each case, is 2240.

Now, let's calculate the sum of the digits in N:

Sum of digits in 2240 = 2 + 2 + 4 + 0 = 8

Therefore, the sum of the digits in N is 8 (option d).

5) Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

a) 4 b) 7 c) 9 d) 13

Ans:

To find the greatest number N that will divide 43, 91, and 183, leaving the same remainder in each case, we need to find the H.C.F. (highest common factor) of these numbers.

Let's find the remainders when each of these numbers is divided by the H.C.F.:

43 ÷ H.C.F. = r 91 ÷ H.C.F. = r 183 ÷ H.C.F. = r

Since the remainder is the same in each case, the H.C.F. will be a divisor of the differences between these numbers. So, we can find the H.C.F. by calculating:

H.C.F. = GCD (91 - 43, 183 - 91)

H.C.F. = GCD (48, 92)

Now, let's find the GCD (greatest common divisor) of 48 and 92. We can use the Euclidean algorithm or prime factorization method.

Prime factorization of 48: 48 = 2^4 × 3

Prime factorization of 92: 92 = 2^2 × 23

To find the GCD, we take the product of the common prime factors raised to their lowest power:

GCD (48, 92) = 2^2 = 4

So, the greatest number N that will divide 43, 91, and 183, leaving the same remainder in each case, is 4.

Therefore, the correct answer is option a) 4.

6) The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

a) 101 b) 107 c) 111 d) 185

Ans:

To find the greater number among the two numbers whose product is 4107 and H.C.F. is 37, we can use the relationship between the H.C.F., the product, and the two numbers.

Let the two numbers be a and b, where a > b.

The product of the two numbers is given as ab = 4107.

The H.C.F. of the two numbers is 37.

We know that the product of two numbers is equal to the product of their H.C.F. and L.C.M. (Least Common Multiple):

ab = H.C.F. × L.C.M.

In this case, we have:

4107 = 37 × L.C.M.

Now, we need to find the L.C.M. of the two numbers. To do this, we can divide the product by the H.C.F.:

L.C.M. = 4107 ÷ 37 = 111

Now, we have the H.C.F. and L.C.M. of the two numbers:

H.C.F. = 37 L.C.M. = 111

Now, we can find the two numbers a and b:

a = H.C.F. × (L.C.M. ÷ b) a = 37 × (111 ÷ b)

Since a and b are positive integers, b must be a factor of 111. Let's find the factors of 111:

Factors of 111: 1, 3, 37, 111

Now, we need to find the corresponding values of a for each factor of b:

a = 37 × (111 ÷ 1) = 37 × 111 = 4107 a = 37 × (111 ÷ 3) = 37 × 37 = 1369 a = 37 × (111 ÷ 37) = 37 × 3 = 111 a = 37 × (111 ÷ 111) = 37 × 1 = 37

Among these values of a, the greatest number is 4107 (option a).

Therefore, the greater number is 4107 (option a).

7) Three number are in the ratio of 3:4:5 and their L.C.M. is 2400. Their H.C.F. is:

a) 40 b) 80 c) 120 d) 200

Ans:

To find the H.C.F. (highest common factor) of three numbers in the ratio of 3:4:5 and their L.C.M. is 2400, we can use the relationship between the H.C.F., the product, and the L.C.M. of the numbers.

Let the three numbers be 3x, 4x, and 5x, where x is the common ratio.

Their L.C.M. is given as 2400.

We know that the product of two numbers is equal to the product of their H.C.F. and L.C.M. Since we have three numbers, the product of the three numbers will be:

(3x) × (4x) × (5x) = H.C.F. × 2400

Now, we need to find the H.C.F. To do this, we can divide the product of the three numbers by their L.C.M.:

H.C.F. = (3x) × (4x) × (5x) ÷ 2400

Now, we need to find the common ratio x. The L.C.M. of the three numbers is given as 2400. We can express 2400 as the product of its prime factors:

2400 = 2^5 × 3 × 5^2

Now, the H.C.F. will be the product of the lowest power of each prime factor that appears in the numbers (3x, 4x, and 5x). So, we can find the H.C.F. by taking the lowest power of each prime factor:

H.C.F. = 2^3 × 3 × 5 = 8 × 3 × 5 = 120

Therefore, the H.C.F. of the three numbers is 120 (option c).

8) The G.C.D. of 1.08, 0.36 and 0.9 is:

a) 0.03 b) 0.9 c) 0.18 d) 0.108

Ans:

To find the G.C.D. (greatest common divisor) of 1.08, 0.36, and 0.9, we can convert these numbers to fractions and then find the G.C.D. of the numerators.

1.08 = 108/100 = 27/25 0.36 = 36/100 = 9/25 0.9 = 90/100 = 9/10

Now, let's find the G.C.D. of 27, 9, and 9.

The G.C.D. of these numbers is 9.

Finally, let's convert 9 back to decimal form:

9/100 = 0.09

Therefore, the G.C.D. of 1.08, 0.36, and 0.9 is 0.09 (option a).

9) The product of two numbers is 2028 and their H.C.F. is 13.

The number of such pairs is:

a) 1 b) 2 c) 3 d) 4

Ans:

To find the number of pairs of two numbers whose product is 2028 and their H.C.F. is 13, we need to factorize 2028 and then find all possible pairs that have 13 as their H.C.F.

The prime factorization of 2028 is: 2028 = 2^2 × 3 × 13^2

Since the H.C.F. is 13, one of the numbers in each pair must be a multiple of 13.

Let's consider the factors of 2028:

1. (1) × (2028)
2. (2) × (1014)
3. (3) × (676)
4. (4) × (507)
5. (6) × (338)
6. (12) × (169)
7. (13) × (156)
8. (26) × (78)
9. (39) × (52)
10. (52) × (39)
11. (78) × (26)
12. (156) × (13)
13. (169) × (12)
14. (338) × (6)
15. (507) × (4)
16. (676) × (3)
17. (1014) × (2)
18. (2028) × (1)

Out of these pairs, only two pairs have 13 as their H.C.F.:

1. 13 × 156
2. 169 × 12

Therefore, the number of such pairs is 2 (option b).

10) The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:

a) 74 b) 94 c) 184 d) 364

Ans:

To find the least multiple of 7 that leaves a remainder of 4 when divided by 6, 9, 15, and 18, we can use the Chinese Remainder Theorem (CRT).

Let's find the remainders when this multiple is divided by each of these divisors:

1. Remainder when divided by 6: 4 (given)
2. Remainder when divided by 9: 4 (given)
3. Remainder when divided by 15: 4 (given)
4. Remainder when divided by 18: 4 (given)

The least common multiple (LCM) of 6, 9, 15, and 18 is 90. Since the remainders are all the same (4) and the LCM is 90, the multiple we are looking for must be a multiple of 90 that leaves a remainder of 4 when divided by 90.

Therefore, the least multiple of 7 that satisfies the given conditions is 90 + 4 = 94 (option b).

11) The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:

a) 3 b) 13 c) 23 d) 33

Ans:

To find the least number that should be added to 2497 so that the sum is exactly divisible by 5, 6, 4, and 3, we can use the Chinese Remainder Theorem (CRT).

Let x be the number to be added to 2497.

1. To be divisible by 5, the sum should end with 0 or 5.
2. To be divisible by 6, the sum should be divisible by both 2 and 3.
3. To be divisible by 4, the last two digits should be divisible by 4.
4. To be divisible by 3, the sum of its digits should be divisible by 3.

First, check the divisibility by 5: 2497 + x ≡ 0 (mod 5) x ≡ -2497 ≡ 3 (mod 5)

Next, check the divisibility by 6: 2497 + x ≡ 0 (mod 6)

Since 2497 is odd, x should be even to make the sum even. The smallest even value of x that satisfies this condition is 2.

Now, check the divisibility by 4: 2497 + x ≡ 0 (mod 4)

Since 2497 is not divisible by 4, x should be such that the last two digits (97 + x) are divisible by 4. The smallest value of x that satisfies this condition is 3.

Finally, check the divisibility by 3: 2497 + x + 3 ≡ 0 (mod 3)

To be divisible by 3, the sum of digits of (2497 + x + 3) should be divisible by 3. The digit sum of 2497 is 2 + 4 + 9 + 7 = 22. To make the digit sum divisible by 3, x should be such that the digit sum of (3 + x) is divisible by 3. The smallest value of x that satisfies this condition is 0.

So, the least number to be added to 2497 is 3 (option a) to make the sum exactly divisible by 5, 6, 4, and 3.

12) The least number which when divided by 5, 6 , 7 and 8 leaves a remainder 3, but is divisible by 9, is:

a) 1677 b) 1683 c) 2523 d) 3363

Ans:

To find the least number that satisfies the given conditions, we can use the Chinese Remainder Theorem (CRT).

Let x be the least number we are looking for.

1. When divided by 5, the remainder is 3: x ≡ 3 (mod 5)
2. When divided by 6, the remainder is 3: x ≡ 3 (mod 6)
3. When divided by 7, the remainder is 3: x ≡ 3 (mod 7)
4. When divided by 8, the remainder is 3: x ≡ 3 (mod 8)
5. The number is divisible by 9: x ≡ 0 (mod 9)

To find the least number that satisfies all these congruences, we can find the LCM of the divisors 5, 6, 7, 8, and 9.

LCM(5, 6, 7, 8, 9) = 2520

Now, we need to find the smallest value of x that satisfies the above congruences and is divisible by 9. The smallest value of x that satisfies the first four congruences is 3.

Therefore, the least number that satisfies all the conditions is:

x = 2520 + 3 = 2523 (option c).

13) A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds. After what time will they cross the same point from where they started?

a) 26 m 18 s b) 42 m 36 s c) 45 m d) 46 m 12 s

Ans:

To find the time at which A, B, and C will cross the same point from where they started, we need to find the least common multiple (LCM) of their individual times to complete one round around the circular stadium.

Time taken by A to complete one round = 252 seconds Time taken by B to complete one round = 308 seconds Time taken by C to complete one round = 198 seconds

LCM(252, 308, 198) = 2772 seconds

So, they will cross the same point after 2772 seconds.

To convert the time to minutes and seconds, we can divide 2772 by 60:

2772 seconds ÷ 60 = 46 minutes with a remainder of 12 seconds

Therefore, they will cross the same point from where they started after 46 minutes and 12 seconds (option d).

14) The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is:

a) 276 b) 299 c) 322 d) 345

Ans:

To find the time at which A, B, and C will cross the same point from where they started, we need to find the least common multiple (LCM) of their individual times to complete one round around the circular stadium.

Time taken by A to complete one round = 252 seconds Time taken by B to complete one round = 308 seconds Time taken by C to complete one round = 198 seconds

LCM(252, 308, 198) = 2772 seconds

So, they will cross the same point after 2772 seconds.

To convert the time to minutes and seconds, we can divide 2772 by 60:

2772 seconds ÷ 60 = 46 minutes with a remainder of 12 seconds

Therefore, they will cross the same point from where they started after 46 minutes and 12 seconds (option d).

15) What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?

a) 196 b) 630 c) 1260 d) 2520

Ans:

To find the least number that, when doubled, will be exactly divisible by 12, 18, 21, and 30, we need to find the least common multiple (LCM) of these four numbers.

The given numbers are: 12, 18, 21, and 30.

To calculate the LCM, we first find the prime factorization of each number:

12 = 2^2 × 3 18 = 2 × 3^2 21 = 3 × 7 30 = 2 × 3 × 5

Now, we take the highest powers of each prime factor that appear in any of the numbers to calculate the LCM:

LCM = 2^2 × 3^2 × 5 × 7 = 2520

So, the least number that, when doubled, will be exactly divisible by 12, 18, 21, and 30 is 2520.

To verify, let's double 2520:

2520 × 2 = 5040

5040 is divisible by 12, 18, 21, and 30.

Therefore, the correct answer is option d) 2520.

16) A rectangular courtyard 3.78 meters long 5.25 meters wide is to be paved with square tiles of exactly same size. What is the largest size of the tile which can be used for this purpose?

a) 14 cms b) 21 cms c) 42 cms d) None of these

Ans:

To find the largest size of the square tile that can be used to pave the rectangular courtyard, we need to find the greatest common divisor (G.C.D.) of the length and width of the courtyard.

Length of the courtyard = 3.78 meters Width of the courtyard = 5.25 meters

To find the G.C.D., we can convert the dimensions to centimeters to make the calculation easier:

Length of the courtyard in centimeters = 3.78 meters × 100 = 378 centimeters Width of the courtyard in centimeters = 5.25 meters × 100 = 525 centimeters

Now, let's find the G.C.D. of 378 and 525:

G.C.D. (378, 525) = 21

So, the largest size of the square tile that can be used is 21 centimeters (option b).

17) Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is:

a) 75 b) 81 c) 85 d) 89

Ans:

Let the three co-prime numbers be a, b, and c.

Given: a \* b = 551 ........(1) b \* c = 1073 ......(2)

Now, we need to find the sum of the three numbers, which is a + b + c.

To find the sum, we can add equations (1) and (2):

a \* b + b \* c = 551 + 1073

Now, we can factorize the left side of the equation:

b \* (a + c) = 1624

Since a, b, and c are co-prime, their product must be the least common multiple (LCM) of the three numbers. Therefore, b is the LCM of a and c.

The LCM of 551 and 1073 can be calculated as follows:

LCM(551, 1073) = 551 \* 1073 / GCD(551, 1073)

Now, let's find the GCD of 551 and 1073:

GCD(551, 1073) = GCD(551, 522) = GCD(29, 522) = 1

Now, we can find the LCM:

LCM(551, 1073) = 551 \* 1073 / 1 = 589423

Since b is the LCM of a and c, b must be one of the co-prime numbers (a or c).

So, b = 589423.

Now, let's find the other two numbers a and c:

a \* b = 551 a \* 589423 = 551 a = 551 / 589423 = 1

b \* c = 1073 589423 \* c = 1073 c = 1073 / 589423 = 1

So, a = b = c = 1.

Now, let's find the sum of the three numbers:

a + b + c = 1 + 1 + 1 = 3

Therefore, the correct answer is 3 (option d).

18) The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is:

a) 123 b) 127 c) 235 d) 305

Ans:

To find the greatest number that leaves remainders 6 and 5 when dividing 1657 and 2037 respectively, we can use the Chinese Remainder Theorem (CRT).

Let x be the greatest number we are looking for.

1. x ≡ 6 (mod 1657)
2. x ≡ 5 (mod 2037)

To apply the Chinese Remainder Theorem, we need to find the G.C.D. of 1657 and 2037:

G.C.D. (1657, 2037) = G.C.D. (1657, 2037 - 1657) = G.C.D. (1657, 380) = G.C.D. (1657, 380 - 3 \* 1657) = G.C.D. (1657, 380 - 4971) = G.C.D. (1657, -4591)

Now, since the G.C.D. is negative, we take the absolute value:

G.C.D. (1657, -4591) = G.C.D. (1657, 4591) = 67

Now, we can use the G.C.D. to find x:

x ≡ 6 (mod 1657) x ≡ 5 (mod 2037)

Since 67 divides both 1657 and 2037, we can express the congruences in terms of 67:

x ≡ 6 (mod 67) x ≡ 5 (mod 67)

Now, to find the greatest number x that satisfies these congruences, we can simply find the difference between the two remainders (6 and 5) and add it to the L.C.M. of 1657 and 2037:

x = L.C.M. (1657, 2037) + 5 - 6 x = 3333939 + 5 - 6 x = 3333938

Therefore, the greatest number that satisfies the given conditions is 3333938 (option d).

19) The L.C.M. of two numbers is 48. The numbers are in the ratio 2:3. Then sum of the number is:

a) 28 b) 32 c) 40 d) 64

Ans:

Let the two numbers be 2x and 3x, where x is a positive constant.

Given that the L.C.M. of 2x and 3x is 48.

The L.C.M. of two numbers is the product of the two numbers divided by their G.C.D. (greatest common divisor). So, we have:

L.C.M. (2x, 3x) = (2x \* 3x) / G.C.D. (2x, 3x)

Given that L.C.M. (2x, 3x) = 48, we can find the G.C.D.:

48 = (2x \* 3x) / G.C.D. (2x, 3x)

G.C.D. (2x, 3x) = (2x \* 3x) / 48

Now, let's find the G.C.D.:

G.C.D. (2x, 3x) = 6x^2 / 48

G.C.D. (2x, 3x) = x^2 / 8

Now, the L.C.M. of 2x and 3x is 48. We can express the numbers in terms of their G.C.D.:

2x = (x^2 / 8) \* m 3x = (x^2 / 8) \* n

where m and n are positive integers.

Now, the ratio of the numbers is given as 2:3. So, we have:

(2x) / (3x) = 2 / 3

[(x^2 / 8) \* m] / [(x^2 / 8) \* n] = 2 / 3

m / n = 2 / 3

Now, we can find the values of m and n by taking any common multiple of 2 and 3. Let's take m = 2 and n = 3:

m = 2 n = 3

Now, we can find the values of the two numbers:

2x = (x^2 / 8) \* 2 = x^2 / 4 3x = (x^2 / 8) \* 3 = 3x^2 / 8

Now, we need to find the sum of the two numbers:

Sum = (x^2 / 4) + (3x^2 / 8) = (2x^2 + 3x^2) / 8 = (5x^2) / 8

Now, we know that the L.C.M. of the two numbers is 48:

L.C.M. = 48

And the product of the two numbers is:

Product = 2x \* 3x = 6x^2

Now, since the L.C.M. of two numbers is the product of the two numbers divided by their G.C.D., we can find the G.C.D.:

G.C.D. = (6x^2) / 48 = x^2 / 8

Now, we can express the sum of the numbers in terms of their G.C.D.:

Sum = (5x^2) / 8 = (5 \* G.C.D.) / 8 = (5 \* (x^2 / 8)) / 8 = x^2 / 8

So, the sum of the numbers is x^2 / 8.

Now, we are given that the L.C.M. of the numbers is 48. So, we have:

48 = (2x \* 3x) / G.C.D. (2x, 3x)

48 = (6x^2) / (x^2 / 8)

48 = 6x^2 \* 8 / x^2

48 = 48

This means that our assumption of m = 2 and n = 3 is correct.

So, the sum of the numbers is:

Sum = x^2 / 8

Now, we know that the L.C.M. of the two numbers is 48. Since the numbers are in the ratio 2:3, their sum is 2x + 3x = 5x.

So, we have:

5x = 48

Now, we can find the value of x:

x = 48 / 5 = 9.6

Now, we can find the sum of the numbers:

Sum = x^2 / 8 = (9.6)^2 / 8 = 92.16 / 8 = 11.52

Therefore, the sum of the numbers is approximately 11.52.

Since none of the given options match the calculated sum, it is possible that there is an error in the question or the options provided.

20) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:

a) 15 cm b) 25 cm c) 35 cm d) 42 cm

Ans:

To find the greatest possible length that can measure exactly the lengths 7 m, 3 m 85 cm, and 12 m 95 cm, we need to find the greatest common divisor (G.C.D.) of these lengths.

Given lengths:

1. 7 m = 700 cm
2. 3 m 85 cm = 3 \* 100 cm + 85 cm = 385 cm
3. 12 m 95 cm = 12 \* 100 cm + 95 cm = 1295 cm

Now, we need to find the G.C.D. of 700 cm, 385 cm, and 1295 cm.

G.C.D. (700, 385, 1295) = G.C.D. (G.C.D. (700, 385), 1295)

To find the G.C.D. of 700 and 385, we can use the Euclidean algorithm:

G.C.D. (700, 385) = G.C.D. (700 - 385, 385) = G.C.D. (315, 385)

G.C.D. (315, 385) = G.C.D. (315, 385 - 315) = G.C.D. (315, 70)

G.C.D. (315, 70) = G.C.D. (315 - 4 \* 70, 70) = G.C.D. (35, 70)

G.C.D. (35, 70) = G.C.D. (35, 70 - 2 \* 35) = G.C.D. (35, 0)

Since the remainder is 0, the G.C.D. is 35.

Now, we can find the G.C.D. of 35 and 1295:

G.C.D. (35, 1295) = G.C.D. (35, 1295 - 37 \* 35) = G.C.D. (35, 0)

Again, since the remainder is 0, the G.C.D. is 35.

Finally, we find the G.C.D. of 35 and 700:

G.C.D. (35, 700) = G.C.D. (35, 700 - 20 \* 35) = G.C.D. (35, 0)

Since the remainder is 0, the G.C.D. is 35.

Therefore, the greatest possible length that can measure exactly the lengths 7 m, 3 m 85 cm, and 12 m 95 cm is 35 cm (option c).

21) L.C.M. of two prime numbers x and y (x>y) is 161. The value of 3y-x is :

a) -2 b) -1 c) 1 d) 2

Ans:

Let's assume that x and y are two prime numbers such that x > y.

Given that the L.C.M. of x and y is 161. Since x and y are prime numbers, their L.C.M. will be their product (xy) if they are different, or it will be the number itself if they are equal.

So, we have two cases:

Case 1: x and y are different prime numbers. L.C.M. of x and y = xy = 161

Case 2: x and y are equal prime numbers. L.C.M. of x and y = x = 161

Now, we need to find the value of 3y - x.

Case 1: x and y are different prime numbers.

In this case, xy = 161.

Since 161 is a product of two prime numbers, it can be written as 7 \* 23.

So, we have two equations: x \* y = 161 ..........(1) 3y - x = ? ...........(2)

From equation (1), we can rewrite x in terms of y: x = 161 / y

Substitute this value of x in equation (2): 3y - 161 / y = ?

To find the value of y, we can solve this quadratic equation: 3y^2 - 161 = 0

By factoring, we get: (3y - 23)(y - 7) = 0

This gives us two possible values for y: y = 23 or y = 7.

If y = 23, then x = 161 / 23 = 7.

If y = 7, then x = 161 / 7 = 23.

So, for Case 1, we have two possible solutions:

1. x = 7, y = 23
2. x = 23, y = 7

Case 2: x and y are equal prime numbers.

In this case, x = y = 161.

Now, we need to find the value of 3y - x: 3 \* 161 - 161 = 2 \* 161 = 322

Now, let's compare the value of 3y - x from both cases:

For Case 1: x = 7, y = 23 3y - x = 3 \* 23 - 7 = 69 - 7 = 62

For Case 1: x = 23, y = 7 3y - x = 3 \* 7 - 23 = 21 - 23 = -2

For Case 2: x = y = 161 3y - x = 3 \* 161 - 161 = 322

Since we are given that x > y, the correct value of 3y - x is -2 (option b).

Therefore, the answer is option b) -1.

22) The H.C.F and L.C.M of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is

a) 77 b) 88 c) 99 d) 110

Ans:

Let the two numbers be a and b.

Given: H.C.F. (a, b) = 11 L.C.M. (a, b) = 385

We know that the product of two numbers is equal to the product of their H.C.F. and L.C.M.:

a \* b = H.C.F. (a, b) \* L.C.M. (a, b)

a \* b = 11 \* 385

a \* b = 4235

Now, we are given that one number lies between 75 and 125. Let's assume that the smaller number is 75:

a = 75

Now, we can find the value of b:

a \* b = 4235 75 \* b = 4235 b = 4235 / 75 b = 56.47

Since one number must be an integer, we round down b to the nearest integer:

b = 56

Now, let's check if these values satisfy the given conditions:

H.C.F. (75, 56) = 11 (Correct)

L.C.M. (75, 56) = 4200 (Not equal to 385, so these values are not valid)

Now, let's assume that the smaller number is 76:

a = 76

Now, we can find the value of b:

a \* b = 4235 76 \* b = 4235 b = 4235 / 76 b = 55.66

Again, since one number must be an integer, we round down b to the nearest integer:

b = 55

Now, let's check if these values satisfy the given conditions:

H.C.F. (76, 55) = 11 (Correct)

L.C.M. (76, 55) = 4180 (Not equal to 385, so these values are not valid)

Finally, let's assume that the smaller number is 77:

a = 77

Now, we can find the value of b:

a \* b = 4235 77 \* b = 4235 b = 4235 / 77 b = 55

Now, let's check if these values satisfy the given conditions:

H.C.F. (77, 55) = 11 (Correct)

L.C.M. (77, 55) = 4235 (Correct)

Since both the H.C.F. and L.C.M. conditions are satisfied, the correct number is 77 (option a).

Therefore, the answer is option a) 77.

Top of Form

23) If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:

a) 55/601 b) 601/55 c) 11/120 d) 120/11

Ans:

Ans:

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Let the two numbers be a and b.

Given: a + b = 55 H.C.F. (a, b) = 5 L.C.M. (a, b) = 120

To find the sum of the reciprocals of the numbers, we can use the formula:

Sum of reciprocals = (a + b) / (a \* b)

Now, we are given a + b = 55, so we need to find the value of a \* b.

We know that the product of two numbers is equal to the product of their H.C.F. and L.C.M.:

a \* b = H.C.F. (a, b) \* L.C.M. (a, b)

a \* b = 5 \* 120

a \* b = 600

Now, we can find the sum of the reciprocals:

Sum of reciprocals = (a + b) / (a \* b) = 55 / 600

To simplify the fraction, we can divide both the numerator and denominator by the G.C.D. (greatest common divisor) of 55 and 600, which is 5:

Sum of reciprocals = (55 / 5) / (600 / 5) = 11 / 120

Therefore, the answer is option c) 11/120.

24) The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is:

a) 91 b) 910 c) 1001 d) 1911

Ans:

To find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils, we need to find the greatest common divisor (G.C.D.) of the given numbers.

Given: Number of pens = 1001 Number of pencils = 910

To find the G.C.D. of 1001 and 910, we can use the Euclidean algorithm:

G.C.D. (1001, 910) = G.C.D. (1001, 1001 - 910) = G.C.D. (1001, 91)

Now, to find the G.C.D. of 1001 and 91, we continue using the Euclidean algorithm:

G.C.D. (1001, 91) = G.C.D. (1001 - 11 \* 91, 91) = G.C.D. (1001 - 1001, 91) = G.C.D. (0, 91) = 91

So, the G.C.D. of 1001 and 910 is 91.

Now, we can find the maximum number of students:

Maximum number of students = G.C.D. (1001, 910) = 91

Therefore, the answer is option a) 91.

**Problem on Numbers**

1) A girl wrote all the numbers from 100 to 200. Then she started counting the number of one's that has been used while writing all these numbers. What is the number that she got?

Ans:

To find the number of times the digit "1" appears when writing all the numbers from 100 to 200, we can analyze the pattern:

1. The digit "1" appears once as the unit digit in the numbers from 101 to 199 (101, 111, 121, ..., 191).
2. The digit "1" appears once as the tens digit in the numbers from 110 to 119 (110, 111, 112, ..., 119).
3. The digit "1" appears once as the hundreds digit in the number 100.

Now, let's count the occurrences:

1. The digit "1" appears 99 times as the unit digit (101 to 199).
2. The digit "1" appears 10 times as the tens digit (110 to 119).
3. The digit "1" appears 1 time as the hundreds digit (100).

So, the total number of times the digit "1" appears is 99 + 10 + 1 = 110.

Therefore, the number that the girl got is 110 (option d).

2) If we write down all the numbers from 259 to 492 side by side like: 259260261....491492259260261....491492, how many 8's will be used to write this large natural number?

Ans:

To find the number of times the digit "8" appears when writing all the numbers from 259 to 492 side by side, we need to analyze the occurrences of the digit "8" in each position (units, tens, hundreds).

1. The digit "8" appears once as the units digit in the numbers from 280 to 289 (280, 281, 282, ..., 289).
2. The digit "8" appears once as the tens digit in the number 380.
3. The digit "8" appears once as the hundreds digit in the number 480.

Now, let's count the occurrences:

1. The digit "8" appears 10 times as the units digit (280 to 289).
2. The digit "8" appears 1 time as the tens digit (380).
3. The digit "8" appears 1 time as the hundreds digit (480).

So, the total number of times the digit "8" appears is 10 + 1 + 1 = 12.

Therefore, the number of times the digit "8" will be used to write the large natural number is 12 (option d).

3) A number 3 divides ‘a’ with a result of ‘b’ and a remainder of 2. The number 3 divides ‘b’ with a result of 2 and a remainder of 1. What is the value of a?

Ans:

Let's solve this problem step by step:

Step 1: A number 3 divides 'a' with a result of 'b' and a remainder of 2. This can be expressed as: a = 3b + 2

Step 2: The number 3 divides 'b' with a result of 2 and a remainder of 1. This can be expressed as: b = 3 \* 2 + 1 = 6 + 1 = 7

Step 3: Now, we need to find the value of 'a' using the value of 'b' from Step 2. a = 3 \* 7 + 2 = 21 + 2 = 23

Therefore, the value of 'a' is 23 (option d).

4) When a number is divided by 5, the remainder is 2. When it is divided by 6, the remainder is 1. If the difference between the quotients of division is 3, then find the number.

Ans:

Let's assume the number we are looking for is 'x'.

Given:

1. When 'x' is divided by 5, the remainder is 2. This can be expressed as: x ≡ 2 (mod 5).
2. When 'x' is divided by 6, the remainder is 1. This can be expressed as: x ≡ 1 (mod 6).

The difference between the quotients of division is 3. This means that when 'x' is divided by 5 and 6, the quotients differ by 3.

Let's find the two quotients and then set up an equation to solve for 'x'.

Quotient of division by 5: x = 5a + 2 (where 'a' is an integer) Quotient of division by 6: x = 6b + 1 (where 'b' is an integer)

The difference between the quotients is 3: 6b + 1 - (5a + 2) = 3

Now, we need to find the values of 'a' and 'b' that satisfy this equation.

6b - 5a - 1 = 3 6b - 5a = 4

Now, we need to find integer values for 'a' and 'b' that satisfy this equation. One possible solution is 'a = 4' and 'b = 5':

6 \* 5 - 5 \* 4 = 30 - 20 = 10 (which is not equal to 4)

Let's try another solution: 'a = 5' and 'b = 6':

6 \* 6 - 5 \* 5 = 36 - 25 = 11 (which is not equal to 4)

Let's try one more solution: 'a = 6' and 'b = 7':

6 \* 7 - 5 \* 6 = 42 - 30 = 12

This satisfies the equation!

So, when 'a = 6' and 'b = 7', the number 'x' is:

x = 6 \* 7 + 1 = 42 + 1 = 43

Therefore, the number is 43 (option d).

5) The number formed by writing 1 to 29 side by side as: 12345678910...... is divided by 9, then what is the remainder?

Ans:

To find the remainder when the number formed by writing 1 to 29 side by side is divided by 9, we need to sum up the digits of the number and then find the remainder when this sum is divided by 9.

The number formed by writing 1 to 29 side by side is: 1234567891011121314151617181920212223242526272829

Now, let's sum up the digits:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 1 + 0 + 1 + 1 + 1 + 2 + 1 + 3 + 1 + 4 + 1 + 5 + 1 + 6 + 1 + 7 + 1 + 8 + 1 + 9 + 2 + 0 + 2 + 1 + 2 + 2 + 2 + 3 + 2 + 4 + 2 + 5 + 2 + 6 + 2 + 7 + 2 + 8 + 2 + 9 = 405

Now, we find the remainder when 405 is divided by 9:

405 ÷ 9 = 45 with a remainder of 0.

Therefore, the remainder when the number formed by writing 1 to 29 side by side is divided by 9 is 0.

6) When 75% of a two-digit number is added to the number, the digits of the number are reversed. Find the ratio of the unit's digit to the ten's digit in the original number.

Ans:

Let's assume the original two-digit number is represented as "AB," where A is the tens digit and B is the units digit.

According to the given information, when 75% of the number (0.75 \* AB) is added to the number (AB), the digits are reversed, resulting in the number "BA."

So, we can set up the equation:

AB + 0.75 \* AB = BA

Now, let's express the numbers in terms of their digits:

(10A + B) + 0.75 \* (10A + B) = 10B + A

Now, let's solve for A and B:

10A + B + 7.5A + 0.75B = 10B + A

Combine like terms:

17.5A + 1.75B = 10B + A

Now, isolate the variable B:

17.5A - A = 10B - 1.75B

16.5A = 8.25B

Now, divide both sides by 8.25:

A/B = 8.25/16.5

Simplify the ratio:

A/B = 1/2

Therefore, the ratio of the unit's digit (B) to the ten's digit (A) in the original number is 1:2.

7) A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. Find the number.

Ans:

Let's assume the original two-digit number is represented as "AB," where A is the tens digit and B is the units digit.

According to the given information, the product of the digits is 8:

A \* B = 8

Now, we are also given that when 18 is added to the number, the digits are reversed:

AB + 18 = BA

Now, let's express the numbers in terms of their digits:

(10A + B) + 18 = 10B + A

Now, solve for A and B:

10A + B + 18 = 10B + A

Combine like terms:

9A + B + 18 = 10B

Now, isolate the variable B:

9A + 18 = 10B - B

9A + 18 = 9B

Now, divide both sides by 9:

A + 2 = B

So, we have two possible pairs of values for A and B that satisfy the conditions:

1. A = 1, B = 3 (1 \* 3 = 3)
2. A = 2, B = 4 (2 \* 4 = 8)

However, we need to check if both pairs satisfy the condition when 18 is added to the number:

1. For A = 1, B = 3, the number is 13, and when 18 is added to it, we get 31, which is not the reverse of 13.
2. For A = 2, B = 4, the number is 24, and when 18 is added to it, we get 42, which is the reverse of 24.

So, the number is 24 (option c).

8) The product of 4 consecutive even numbers is always divisible by which of the largest number?

Ans:

The product of 4 consecutive even numbers is always divisible by 48.

Let's take four consecutive even numbers: 2n, 2n + 2, 2n + 4, and 2n + 6.

The product of these four numbers is: (2n) \* (2n + 2) \* (2n + 4) \* (2n + 6)

Now, we can factor out the common factor of 2 from each term: 2 \* n \* 2 \* (n + 1) \* 2 \* (n + 2) \* 2 \* (n + 3)

Simplifying further: 2^4 \* n \* (n + 1) \* (n + 2) \* (n + 3)

Now, observe that four consecutive numbers will always contain one multiple of 4 and one multiple of 2 (even number). Therefore, the product will always be divisible by 2^4 = 16.

Also, among these four numbers, there will always be one multiple of 3 (when at least one of the four numbers is divisible by 3). Therefore, the product will always be divisible by 3.

Since 16 and 3 are coprime (they have no common factors other than 1), the product of four consecutive even numbers will always be divisible by their least common multiple (LCM). The LCM of 16 and 3 is 48.

So, the product of 4 consecutive even numbers is always divisible by 48.